1) Evaluate $\int_C \bar{z}\, dz$ for the ~~foo~~ llowing paths

(a) $C: 0 \to 3+9j$ along $y = x^2$

(b) $C:$ ~~unit~~ unit circle

2) let $f(w) = \oint \frac{z^2 + 6z - 2}{z - w}\, dz$ where $C: |z| = 4$

Determin $f(1+j)$

3) $f(z) = \frac{6z^2 + (1-3j)z - 3 - j}{(z^2 - 1)(z - j)}$

Evaluate $\oint_C f(z)\, dz$ where C is the circle $|z - 1 - j| = 1.5$

4) Select an integration path C from $z = 1$ to $z = 1 + j$ and evaluate $\oint_C \sin(z)\, dz$

justify your path selection ie reasoning for selecting the path and write your answer in the form of $a + jb$

1) $\oint \bar{z}\, dz$

a) C $\quad 0 : 3 + j9 \quad$ along $y = x^2$

b) C $\quad$ unit circle

1) $\bar{z}$ not analytic

a) $x = t \qquad 0 \le t \le 3$

$y = t^2$

$z(t) = t + jt^2$

$\oint \bar{z}\, dz = \oint f(z(t)) \cancel{dt}\ \dot{z}(t)\, dt$

$$\oint \bar{z}\, dz = \oint_{t_1}^{t_2} \overline{z(t)} \cdot \dot{z}(t)\, dt$$

$\overline{z(t)} = t - jt^2$

$\dot{z}(t) = 1 + 2jt$

$$= \int_0^3 (t - jt^2)(1 + 2jt)\, dt$$

$$= \int_0^3 t \ \cancel{+t^2} j + 2t^3\, dt$$

$$= \frac{1}{2} t^2 \Big]_0^3 + \left(2 \cdot \frac{1}{4}\right) t^4 \Big]_0^3 + j \cdot \frac{1}{3} t^3 \Big]_0^3$$

$$= \frac{1}{2}[3^2 - 0] + \frac{1}{2}(3^4 - 0) + \frac{j}{3}(3^3 - 0)$$

$$= 9/2 + 81/2 + \frac{27}{3} j$$

$$\oint_c \bar{z}\, dz = \boxed{45 + 9j}$$

b) $\oint \bar{z}\, dz$ on unit circle

$$\cos t + j \sin t = e^{jt} = z(t)$$

$$0 \leq t \leq 2\pi$$

$$\overline{z(t)} = e^{-jt}$$

$$\dot{z}(t) = j e^{jt}$$

$$\oint_0^{2\pi} j e^{jt} \cdot e^{-jt}\, dt$$

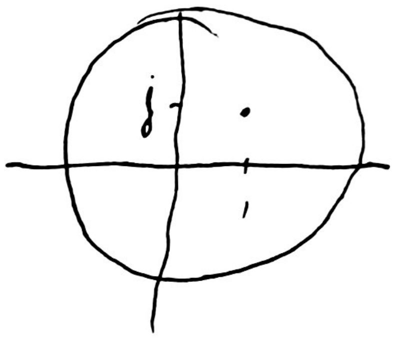
$$j \oint_0^{2\pi} dt$$

$$\oint_c \bar{z}\, dz = \boxed{2\pi j}$$

2) let $f(w) = \oint_c \frac{z^2+6z-2}{z-w}\,dz \qquad c : |z| = 4$

Determine $f(\underset{\sim}{1+j})$

$$f(1+j) = \oint \frac{z^2+6z-2}{z-(1+j)}\,dz$$



$$\oint \frac{f(z)}{z-z_0}\,dz = 2\pi j\, f(z_0)$$

$$f(z) = z^2+6z-2$$

$$z_0 = 1+j$$

$$f(z_0) = (1+j)^2 + 6(1+j) - 2$$

$$= 4+8j$$

$$\oint \frac{z^2+6z-2}{z-(1+j)} = 2\pi j\,(4+8j)$$

$$= \boxed{-16\pi + 8\pi j}$$

3) Given $f(z) = \dfrac{6z^2 + (1-3j)z - 3 - j}{(z^2-1)(z-j)}$

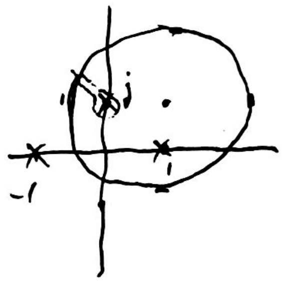
$$\oint \frac{f(z)}{z - z_0} = 2\pi f(z_0)$$

$\int_C f(z)\,dz$ where C $|z-1-j| = 1.5$

$z^2 - 1 = 0$

$z = \pm 1$

$z = j$



$$\oint_c \frac{6z^2 + (1-3j)z - 3 - j}{(z^2-1)(z-j)} = \int \frac{A}{(z-j)} + \int \cancel{\frac{B}{(z+1)}}^{\,0} + \int \cancel{\frac{C}{(z-1)}}$$

$$6z^2 + (1-3j)z - 3 - j = \frac{A}{z-j} + \frac{B}{z+1} + \frac{C}{z-1}$$

$$= A(z^2-1) + B(z-j)(z-1) + C(z-j)(z+1)$$

$6(j)^2 + (1-3j)\cdot j - 3 - j \doteq A$

$\cancel{-6 + j + 3 - 3 - j}$

j

$6(j)^2 + (1-3j)\cdot j - 3 - j = A(j^2-1)$

$-6 + \cancel{j+3-3-j} = -2A$

$\frac{-6}{2} = \frac{-2A}{-2}$

$A = 3$

$$1$$

$$6(1)^2 + (1-3j)(1) + -3 - j = C((1-j)(1+1))$$

$$6 + 1 - 3j - 3 - j = (2-2j)C$$

$$\frac{4-4j}{2-2j} = \frac{(2-2j)C}{2-2j}$$

$$2 = C$$

$$-1$$

$$6(-1)^2 + (1-3j)(-1) - 3 - j = B(-1-j)(-1-1)$$

$$6 + -1 + 3j - 3 - j = (2+2j)B$$

$$\frac{2+2j}{2+2j} = \frac{(2+2j)B}{(2+2j)}$$

$$1 = B$$

$$\oint \frac{6z^2(1-3j)z-3-j}{(z^2-1)(z-j)} = \int \frac{3}{z-j} + \int \cancel{\frac{1}{z+1}} + \int \frac{2}{z-1}$$

$$3\underbrace{\int \frac{1}{z-j}\,dz}_{=2\pi j} + 2\underbrace{\int \frac{1}{z-1}\,dz}_{=2\pi j}$$

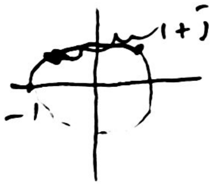
$$3 \cdot 2\pi j \cdot + 2 \cdot 2\pi j$$

$$6\pi j + 4\pi j = \boxed{10\pi j}$$

4) C: $z_1 = -1$ $z_2 = 1+j$

$$\int_c \sin z \, dz$$

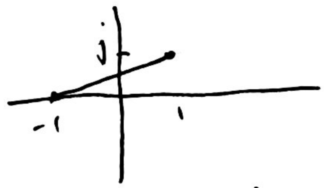
Select a path and jusify why picked that path



$\sin(z)$: analytic

~~$\cos t + j \sin(t) = e^{jt}$ $-\pi \le t \le \frac{\pi}{4}$~~

~~justification: Using π for parameterizing the path makes the integration e~~



$$\int \sin(z)\, dz = -\cos(z)\, dz$$

$$y = \frac{j}{2} x + \frac{j}{2}$$

$$-\cos(z_2) + \cos(z_1)$$

$x = t$ $t = -1 \le t \le 1$

$$F(b) - F(a)$$

$$y = \frac{j}{2} t + \frac{j}{2}$$

$$-\cos(1+j) + \cos(-1)$$

$$x = t \quad -1 \le t \le 1$$

$$-.29 + .98j$$

$$y = \frac{j}{2} t + \frac{j}{2}$$